The Geometry of Mayburgh Henge

In the story “Sherlock Holmes in Cumbria” Dr. Watson visits the ancient monument of Mayburgh Henge south of the town of Penrith.

Watson correctly avoids any calculations about the geometry of the henge – itself a misnomer, there are no “hanging stones” as in the famous structure on Salisbury plain, but the term “henge” has transferred from Stone Henge to circular monuments involving stones regardless of the configuration of the stones; or in this case the single remaining stone of the Mayburgh.

The shape can be considered as approximately half a torus – a doughnut shaped object – in this case a circular ring where the boundary of the ring is a mound with a half circular cross section I.e. – a doughnut sliced in half.

The shape is in fact a “volume” of revolution: a torus is the shape of circle when sliced through, and then that circle rotated in a circle to generate a larger ring.

Watson provides the inner radius, i.e. the distance from the centre to the start of the encircling bank, and an estimate of the width of the bank that was surmounted by trees. These numbers are 60 yards, 10 yards, so the outer size of the structure (rising from the flat field) is 80 yards across with an outer perimeter of $2 \times \pi \times 40$ yards.

The calculation of the volume involves integral calculus, but it was solved for torus long before this mathematics was discovered (or invented!). Archimedes knew the solution before 1 B.C.E.

If the inner radius or 60 yards is “r” and the outer radius is 60 + 10 yards in “R” the formula for the volume of such a torus is:

$$0.25 \times \pi^2 \times (R + r) \times (R - r)^2$$

Alturnately measuring from the centre of the torus to the centre of the bank is 65 yards (A), and the radius of the bank is 5 yards (B), the calculation is:

$$2 \times \pi^2 \mathbf{x} A \times B^2$$

If we write this differently as

$$(2 \times \pi \times A) \times (\pi \times B^2)$$

we see that this is the circumference of large circle (the henge) multiplied by the area of the cross section of the area for the encircling mound (that is if the Mayburgh were a torus, rather than one half of a torus).

If the cross section of the encircling mount were half a circle this would be too large since the mound does not rise vertically from the flat field. The cross section from the field would be something like a Bell Curve – gradually rising then forming a flatish top before descending again.

If the cross section of the mound were a half a circle, the area would be ½ x π x B2. And if it were a triangle consisting of two smaller right angle triangles of base B and height B placed back to back, the area would be

$$\frac{1}{2} \times B \times B + \frac{1}{2} \times B \times B$$

in other words B^2.

The cross section is in fact something the area of a Bell Curve shaped figure, or the shape of a Sine curve from trough to trough; and this area approximates to that of two back to back right angle triangles, or B^2.

thus the volume of the Mayburgh is

$$\frac{1}{2} \times ((2 \times \pi \times A) \times (B^2))$$

0.5 * ((2 * 3.14159 * 65) * (5 * 5))

= 5,105.08 cubic yards

This number is in cubic yards, so coverting to metric litres yields 3,903,113.71 litres.

A house brick (at least here in Australia) is 23 x 11 x 7 cms or 1771 cm^3 (or 1.771 litres) and weighs about 2.1 kilograms.

So if we assume the henge were mostly built of rocks (similar to a house brick), that means that the volume of the Mayburgh ring involves the moving the equivalent of some 2.2 million (2,203,904) bricks, or roughly 4.6 megatonnes of earth and rock, that is about 5 million US tons.

NOTE

Watson's measuring is inaccurate being in paces and guesswork. The ring forming the henge is not uniform height, and the gap in it that forms the entrance is not excluded from the calculations.

Nevertheless the population of the surrounding area to the Mayburgh for a radius of 10 km is at the time of writing is about 30,000, but given that the henge was build maybe 4,000 (or more) years ago, then the local nearby population could easily be not 30,000 but more like 2,000. So the effort and time to build the Mayburgh was considerable, especially since there are many other ancient earthworks and henges nearby.

Thus the construction of the Mayburgh involved a substantial amount of effort from the local population. And where did the building material come from? There are no excavations or holes to explain where the 4 megalitres of material came from.